the position of a ship had been located several miles inland by several practised observers.

Captain Carpenter, R.N., referred to the high degree of accuracy to be obtained by means of a small sextant, by making many series of observations and taking the mean.

Naval Instructor Ainslie announced that the Council had sanctioned the formation of an "Instrument Section," for the purpose of collecting information on the use and construction of instruments, and on the best ways of dealing with difficulties that might occur in their use.

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**The Pulsation Theory of Cepheid Variables.**

Although the variable stars of the Cepheid type show a periodic change of radial velocity, the opinion has recently gained ground that they are not binary stars. The cause of the light-variations has always been perplexing*; they are not accounted for by geometrical considerations; and explanations based on tidal disturbances or motion in a resisting medium have not met with much success. A strong case against the binary hypothesis has been made out by H. Shapley †; one important consideration is that the radius that must be ascribed to the orbit is usually much less than the radius of the star itself, so that it is difficult to find room for the hypothetical companion.

According to the alternative theory, the variations are due to pulsations of a single star, which cause changes in the rate of emission of light. There seems no reason to doubt the reality of the observed radial velocities, so we may assume that they indicate the rate of motion of the surface, or the most luminous part of it, in the course of the pulsation. As the simplest type of oscillation, the star may be supposed to expand and contract symmetrically. I am not sure whether this will be found consistent with the observations, since it would seem that the spectral lines should be broadened at times of maximum velocity, the different parts of the surface having different component motions in the line of sight. However, any other type of pulsation will lead to very similar conclusions.

The double amplitude of the pulsation \(2a \sin i\) averages for the different stars about 2,000,000 km., the greatest yet found being 4,000,000 km. The Cepheids are giant stars of low density with radii much greater than that of the Sun; so there is nothing inadmissible in such large changes of radius, but the consequent internal changes in the star must be very far-reaching. This change of volume will cause some light-variation apart from any changes of temperature; but it is evidently not the main cause

* A summary has been given by D. Brunt, *Observatory*, vol. xxxvi. p. 59.
of Cepheid variation, since it is found that the greatest contrast is between the times when the star is expanding and contracting most rapidly—i.e., when it is passing through the mean volume.

The most difficult question is, how can these pulsations be maintained? It is suggested by Shapley that, if the pulsations were started by some cataclysm, there is one type which would decay extremely slowly; it might persist almost indefinitely with inappreciable dissipation. But I do not think this conclusion is warranted by such investigations as have been made. The problem is essentially a thermodynamical one. The main cause likely to lead to a decay of vibration is thermal dissipation of energy due to the flow of heat between different parts of the star during the prodigious changes of temperature in the interior, of which the observed surface-changes are probably only a faint
index.

If the pulsation theory is correct, this tendency to dissipate the energy of vibration must be counteracted in some way. The lost energy must be replaced from some other source *. There is an evident supply of energy which might be utilized, since heat is continually liberated within the star and passes outward into space; this may be borrowed and converted into energy of pulsation. But, in order to convert heat of any kind into work, the star, or some part of it, must behave as an engine in the thermodynamical sense: that is to say, it must take in heat when it is at a higher temperature than the average and give out heat at a lower temperature—just the opposite to what usually happens in natural conditions. An apparent exception to this rule must be noted. The heat of the star is flowing outwards, and is more “available” for conversion into work than if it flowed in all directions indiscriminately; it is in an intermediate state between mechanical and totally degraded energy, and by means of radiation-pressure a portion of the energy can be recovered in mechanical form.

The study of the vibrations of a star, taking into account the flow of heat, is a very difficult analytical problem; and it has not yet been possible to find the conditions under which the star could behave in the manner of an engine. Possibly during the pulsation, variations of the transparency, which governs the flow of heat, might cause the engine to be fed in the required manner. We must be careful not to prove too much—every star is not a Cepheid variable. Though we cannot offer any adequate theory as to how the star manages to behave as an engine, we can point out some evidence that it does so behave. I am not sure whether the following mode of regarding the question is strictly allowable; but I venture to put forward the suggestion tentatively.

Consider the mode in which thermal dissipation acts in the

* It is convenient to speak as though energy were actually lost by one cause and gained by another cause; but we are, of course, dealing with two opposing tendencies rather than actually separable processes.
case of a sound-wave. The air is hottest at a point of condensation. If this heat leaks away, the compressed air loses some of its spring, and the expansion which follows has diminished energy—consequently, the waves decay. If, on the other hand, the air could be persuaded to lose heat at points where it was rarefied and coolest, the ensuing compression would be assisted and the waves reinforced. Intermediately, a loss or gain of heat at a point of normal density neither dissipates nor increases the energy. The decay or reinforcement of the vibration thus depends on the difference of phase between the density-changes and the changes of emission of heat; except, however, under special—almost artificial—conditions the phase-relation is always such as to produce decay. But in a Cepheid star we have very nearly the intermediate case above-mentioned; it is a well-known phenomenon that these variables radiate most intensely at the time of maximum velocity of approach, and least when receding: in both cases the star must then be passing through its mean state of density. Considering excess or defect from the average rate of outflow, we may say that the material relatively loses heat as it passes through its normal density expanding, and gains heat at the same stage contracting; but neither gains nor loses when most compressed or most rarefied. Approximately, therefore, the waves neither decay nor increase; the natural tendency to decay has been in some way opposed and neutralized.

It would be difficult to estimate how accurate a balance is indicated by the observational data, since the oscillations of a Cepheid are not strictly simple harmonic. There is, however, another cause which must tend to maintain the pulsations in the face of any residual dissipation. Since the outflow of radiation is greatest when the star is expanding, light-pressure is then especially strong and assists to drive the material outwards; during contraction, radiation is below the average and the material flows back against a diminished light-pressure.

It will be seen that the condition for absence of dissipation is satisfied when the light-maximum corresponds to maximum velocity either of approach or recession; but radiation-pressure only assists when it corresponds to approach.

The observed flow of heat is thus in the appropriate phase to maintain or increase the pulsation. How this comes about must be left unsolved; but, since it is so, it seems clear that the pulsations are likely to be maintained. We may go a step further, and assume that, if an infinitely small pulsation were started, it would be gradually increased by the causes discussed until it grew to the large disturbance which we observe. There is then no need to postulate a cataclysm to originate the variation. In that case what limits the increase? When we have, as in this case, two opposing tendencies, there may be some limiting amplitude for which they just balance; for any further increase of amplitude, dissipation would exceed the reinforcing cause. But there is one evident upper limit to the motion, which may or may
not be the effective limit. When the contraction is about to take place, the acceleration of the surface cannot possibly exceed the acceleration due to gravity, since no other inward force is acting. This condition is easily satisfied in all the Cepheids I have examined—with such a margin, indeed, as to make it doubtful whether it has anything to do with the stoppage. I suppose a typical Cepheid might have a semi-amplitude of 1,000,000 km. and a period of 6 days. Taking the motion to be simple harmonic, the maximum acceleration is then 14 cm./sec.\(^2\). Gravity at the surface of an ordinary giant star of type G is probably about 300 cm./sec.\(^2\), and it is not until we get down to the density usually associated with type M that we reach a value of gravity so low as 14 cm./sec.\(^2\). Estimates of the densities of Cepheid variables have been made from their absolute luminosities, and also (more hypothetically) from their periods of pulsation; the two modes of estimate are in good agreement, and seem to show that the densities are considerably less than those of average stars of the same type. But, after making all allowances, I think the maximum acceleration scarcely reaches a tenth of the value of gravity at the surface. However, the order of magnitude is sufficiently near to suggest that the limit of pulsation is in some way related to this condition (which is essentially that the gaseous pressure cannot become negative); and, perhaps, if we took into account the electrical forces and the radiation-pressure, which may be resisting gravity, a satisfactory explanation would be reached.

A. S. Eddington.

The Sun-spots of January and February 1916.

As mentioned on p. 162 of the Observatory for April 1917, it has been determined not to give a series-number in future to very small groups or isolated spots unless they are observed upon more than one day. Such markings, when their actuality has been established by their detection on two or more photographs taken on the same day, will be measured and reduced, and their areas and heliographic co-ordinates will eventually be published in the 'Greenwich Photo-Heliographic Results.' Here they will appear under the number of the rotation to which they belong, and for purposes of reference will be further distinguished by an identification letter. Thus the four "ephemeral spots" numbered on p. 165 as Groups 7583, 7587, 7589, and 7590 respectively, being the first four markings of the kind to be registered in Rotation 833, will be registered as R 833\(a\), R 833\(b\), R 833\(c\), and R 833\(d\). In the meantime, these markings, being at once so small and short-lived and usually so faint, will be omitted from this series of lists published in the Observatory, and will only be referred to in cases of special interest.

In January 1916 the "ephemeral groups" registered were 21 in number—2 on Greenwich photographs, 18 on those taken at