Physical Conditions in the Initial Stages of the Expanding Universe*,†

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The detailed nature of the general nonstatic homogeneous isotropic cosmological model as derived from general relativity is discussed for early epochs in the case of a medium consisting of elementary particles and radiation which can undergo interconversion. The question of the validity of the description afforded by this model for the very early super-hot state is discussed. The present model with matter-radiation interconversion exhibits behavior different from non-interconverting models, principally because of the successive freezing-in or annihilation of various constituent particles as the temperature in the expanding universe decreased with time. The numerical results are unique in that they involve no disposable parameters which would affect the time dependence of pressure, temperature, and density.

The study of the elementary particle reactions leads to the time dependence of the proton-neutron concentration ratio, a quantity required in problems of nucleogenesis. This ratio is found to lie in the range $\sim 4.5:1-\sim 6.0:1$ at the onset of nucleogenesis. These results differ from those of Hayashi mainly as a consequence of the use of a cosmological model with matter-radiation interconversion and of relativistic quantum statistics, as well as a different value of the neutron half-life.

I. INTRODUCTION

THE nonstatic homogeneous isotropic cosmological model which satisfies the equations of general relativity has received a great deal of attention. However, the detailed nature of the model does not appear to have been examined at the extremely high temperatures and densities characteristic of the very early stages of the expanding universe. This question has been examined in the present paper and the dependence of the temperature and density on time has been determined for the case where the radiation density (taken to include photons, neutrinos, electrons, positrons, and mesons) is much greater than the density of matter (nucleons). For initial conditions compatible with present astrophysical observations, one can demonstrate that the radiation density exceeded the density of matter for about the first hundred million years in the expansion.

We have carried our study of this problem back to a temperature of $\sim$100 Mev ($\sim 1.2\times 10^{12}$°K), corresponding to an epoch of $\sim 10^{-4}$ sec. For temperatures below this value one can treat reactions among elementary particles with some confidence. Furthermore, below $\sim$100 Mev the energy stored in the gravitational field is a negligible part of the total energy so that the question of using a correct unified field theory, including the quantization of the field equations, can be avoided. Finally, at $\sim$100 Mev one has a state of thermodynamic equilibrium among all the known constituent particles and radiation so that a knowledge of the previous history of the universe is not required. As part of the detailed study of the cosmological model we have examined the reactions among the elementary particles present and followed their course in the universal expansion. As will be seen, all reaction rates, except those involving the neutrino, are sufficiently high to maintain thermodynamic equilibrium. An examination of the kinetics of the reactions between nucleons and neutrinos has yielded the relative concentrations of protons and neutrons as a function of time. The only parameters involved in the cosmological model are the nucleon density and radius of curvature. At the early times prior to element formation, neither of these parameters affects the course of events because of the very high total density and because the nucleon density is neglible compared with the density of radiation. The nucleon density becomes of importance at later times in considering element formation, while the radius of curvature becomes of interest only at times of the order of a hundred million years.

The foregoing detailed considerations of the early stages of the universal expansion bear significantly on the neutron-capture theory of element formation. This theory has been concerned principally with understanding the general trend in the distribution of the cosmic abundances of the chemical elements with atomic weight.[1,2]

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[1] R. A. Alpher and R. C. Herman, Revs. Modern Phys. **22**, 153 (1950); Phys. Rev. **84**, 60 (1951).

[2] R. A. Alpher and R. C. Herman, *Annual Review of Nuclear Science* (Annual Review of Nuclear Science, Inc., Stanford, 1953), vol. 2, p. 1. The simple-neutron capture theory has satisfactorily reproduced for all but the lightest elements the observed approximately exponential decrease in abundance with increasing atomic weight up to $A\sim 100$, as well as the approximate constancy of abundance for the heavier elements. Briefly, in the neutron-capture theory, as thus far developed, the various nuclear species were supposed to have been formed from nucleons by the successive radiative capture of fast neutrons with adjustment of nuclear charge by intervening β decay during the early stages of the expansion of the universe. The primordial material or ylem was taken to be a mixture of neutrons and radiation. As the universal expansion proceeded the neutrons underwent free decay, so that by the time the universal temperature had decreased to a value

For the lightest elements the processes of neutron capture and β decay, while adequate to explain the formation of the heavier elements, must be supplemented by thermonuclear reactions involving protons, deuterons, and other light nuclei. The very light element reactions were examined in some detail by Fermi and Turkevich,[1] using the cosmological model previously employed for the neutron-capture theory, with a finite starting time and a primordial mixture of neutrons and radiation. This improved light-element calculation did not satisfactorily resolve what remains the principle difficulty of the theory, namely, the deduction of the specific nuclear reactions and physical conditions which might carry the formation chain of reactions through and beyond atomic weight 5. To resolve this and other difficulties in the theory, it will apparently be necessary to remove many of the simplifying restrictions. In particular, the assumption of a starting time must be replaced by detailed consideration of element-building reactions increasing in importance from very early times in the universal expansion as the rates of various dissociative processes diminish with decreasing temperature. Moreover, one should include all possible reactions among the elementary particles, since these reactions, which are important at very high temperatures, may influence the physical conditions that control the element-building processes.

The elementary-particle reactions determine the ratio of the relative concentrations of protons and neutrons, a quantity which plays a vital role in predicting the general trend of abundances according to the neutron-capture theory. As has already been mentioned, in previous calculations the proton-neutron abundance ratio has been taken to be that resulting from free decay of the primordial neutrons during the period from the start of the expansion up to the starting time selected for element-building reactions. A more detailed calculation was made by Hayashi,[3] who determined the value of the proton-neutron ratio resulting from spontaneous and induced β processes among protons and neutrons in the presence of electron pairs and neutrinos in the early stages of the expansion. Whereas on the basis of the crude assumption of neutron decay only, one obtains a proton-neutron ratio of $\sim$1:7, Hayashi's calculation gave $\sim$4:1 by the starting time for element-building reactions. With this latter value of the ratio, it has not yet proven possible to represent the cosmic abundance distribution in atomic weight on the basis of the simple neutron-capture theory,[2] a theory which contains only one arbitrary parameter, *viz.*, the density of matter at the start of the element-building epoch, and which involves only neutron-capture reactions. In part because of this difficulty and because it seemed worth while to investigate the effect of certain modifications on Hayashi's calculation of the final value of the proton-neutron ratio, the work described in the remainder of this paper was carried out. Among the changes involved in the present study are the use of relativistic quantum statistics instead of Boltzmann statistics, a modified cosmological model for early epochs as required by the interconversion of matter and radiation, which as we have already indicated is of considerable interest for its own sake, and the use of the value of the neutron half-life recently reported by Robson[4] which differs materially from the older value employed by Hayashi.

It seems most likely that element synthesis is intimately connected with questions of cosmology. In the present work we consider the sequence of events up to the time when the rate of element formation became significant. As we shall see later in detail, all the constituents remained in thermodynamic equilibrium as the universe expanded and cooled to a temperature of $\sim$10 Mev. At $\sim$10 Mev the neutrinos were essentially frozen out of the equilibrium. By $\sim$0.3 Mev the proton-neutron ratio was almost entirely determined by the free decay of the neutron. It remains for future study to re-examine the formation of the elements by thermonuclear reactions as a subsequent part of the picture developed here. A detailed chronology is given in a later portion of this paper [see Sec. V].

II. THE COSMOLOGICAL MODEL

The theory of element formation by non-equilibrium thermonuclear reactions has been developed as an integral part of the very early stages of the expanding universe. Detailed calculations of the necessary rate processes require a knowledge of the temporal behavior of temperature, density, and rate of expansion during these early epochs. The cosmological model that has been used previously for this purpose is the most general nonstatic model satisfying the requirements of general relativity, exhibiting homogeneity and isotropy, and which is composed of a perfect fluid with no interconversion of matter and radiation.[1,2] The rate of expansion and, implicitly, the rate of change of temperature in the expansion for this model, with no restrictions on the composition of the perfect working fluid, are given in relativistic units by the following differ-

where nuclei would be thermally stable, an appreciable number of protons had been generated. Then the capture of neutrons by protons provided the first step in the formation of the successively heavier elements. More specifically, the temperature for the beginning of building-up reactions was taken to be $\sim$0.1 Mev (corresponding to a specific starting time for element building in the cosmological model used, in which $T=1.52\times10^{10}t^{-\frac{1}{2}}$°K). This choice was dictated by the magnitude of the binding energy of the deuteron on the one hand and by the lack of evidence in the abundance data for any resonance neutron capture on the other hand. At the starting time, neutron decay had led to a proton-neutron ratio of $\sim$1:7.

One of the approximations involved thus far in calculations with the neutron-capture theory has been the smoothing of available data on fast neutron radiative capture cross sections as a function of atomic weight. Moreover, reactions other than radiative neutron capture among the very lightest elements have been ignored.

[3] C. Hayashi, Progr. Theoret. Phys. (Japan) 5, 224 (1950).

[4] J. M. Robson, Phys. Rev. 83, 349 (1951).

ential equations:[5]

$$-\frac{e^{-g}}{R_0^2}-\frac{d^2g}{dt^2}-\frac{3}{4}\left(\frac{dg}{dt}\right)^2+\Lambda=8\pi p_0, \tag{1a}$$

$$\frac{3e^{-g}}{R_0^2}+\frac{3}{4}\left(\frac{dg}{dt}\right)^2-\Lambda=8\pi\rho_{00}, \tag{1b}$$

and

$$e^{\frac{1}{2}g(t)}=l/l_0=R/R_0, \tag{1c}$$

where l and R are proper distance and radius of curvature, respectively, given in units of l_0 and R_0, Λ is the cosmological constant, and p_0 and ρ_{00} are proper pressure and density. The quantities p_0 and ρ_{00} are functions of temperature and of l, and hence implicitly of time. Equation (1b) may also be rewritten, by using Eq. (1c), in the following form:

$$\frac{dl}{dt}=+\left(\frac{8\pi}{3}\rho_{00}l^2-\frac{l_0^2}{R_0^2}+\frac{\Lambda l^2}{3}\right)^{\frac{1}{2}}, \tag{2}$$

with the plus sign taken to indicate expansion. We have taken $\Lambda=0$ in keeping with current practice.[6] As can be easily shown, the constant term l_0^2/R_0^2 in Eq. (2) may be neglected in the application of this model to early epochs. This is equivalent to neglecting l^2e^{-g}/R_0^2 in Eq. (1). If $\rho_{00}\propto l^{-n}$ where $n>2$, then for sufficiently early times l will be so small that one has $8\pi\rho_{00}l^2/3 \propto 8\pi l^{2-n}/3 \gg l_0^2/R_0^2$. Hence, for early epochs in the expansion one may replace Eq. (2) by

$$\frac{dl}{dt}=\frac{l}{2}\frac{dg}{dt}=+\left(\frac{8\pi}{3}\rho_{00}l^2\right)^{\frac{1}{2}}. \tag{3}$$

As has already been mentioned, the cosmological model, which is discussed in this paper, taken together with the presently observed smoothed-out matter density in the universe as well as the estimated age, are consistent with the supposition that during the early epochs of interest the matter density was much smaller than the radiation density (i.e., $\sim 1:10^6$). The neutron-capture theory of element formation[1] requires that the radiation density greatly exceeded the density of matter during the early epochs of the universal expansion. In this previous work it was not necessary to consider the interconversion of matter and radiation since for the epochs considered the temperature was already below that required to maintain a significant electron-pair density. Hence, the working fluid for the cosmological model was taken as black-body radiation, containing a trace of matter, and expanding adiabatically according to $T\propto 1/l$. It has been shown[1] that for early epochs Eq. (3) leads to the following expressions for the radiation density, ρ_γ, the matter density, ρ_m, the total density, ρ_{total}, the temperature, T, and proper distance, l, with $\rho_\gamma \gg \rho_m$:

$$\rho_{\text{total}} \cong \rho_\gamma \cong [3/(32\pi G)]t^{-2}=4.48\times 10^5 t^{-2}\ \text{g/cm}^3, \tag{4}$$

$$\rho_m=\rho_0 t^{-\frac{3}{2}}\ \text{g/cm}^3, \tag{5}$$

$$T=(c^2\rho_\gamma/a_\gamma)^{\frac{1}{4}}=1.52\times 10^{10}t^{-\frac{1}{2}}\,{}^\circ\text{K}, \tag{6}$$

and

$$l=(32\pi G\rho_{\gamma''}l_0^4/3)^{\frac{1}{4}}t^{\frac{1}{2}}, \tag{7}$$

where G is the gravitational constant, c is the velocity of light, a_γ is the Stefan-Boltzmann constant, t is the time in seconds from the "start" of the expansion, $\rho_{\gamma''}$ is the density of radiation when $l=l_0$, and ρ_0 is a constant. As will be seen, the above equations are still valid in the case discussed in the present paper providing that $t \gtrsim 100$ sec, ρ_γ is eliminated from Eq. (4), the constant 1.52 in Eq. (6) becomes 1.45 for Majorana neutrinos and 1.38 for Dirac neutrinos, and $\rho_{\gamma''}$ in Eq. (7) is replaced with the value of ρ_{total} when $l=l_0$. The quantity ρ_0 is the one arbitrary parameter in the simple neutron-capture theory. It has been adjusted in previous calculations[2] so that the density of matter at the start of the element-forming processes would lead to the observed cosmic abundance distribution. The cosmological model at early epochs described by Eqs. (4) and (6) was adopted by Hayashi as a basis for his calculation of the proton-neutron ratio.

While we have assumed a homogeneous and isotropic model of the universe in agreement with present astronomical evidence, it should be pointed out that this restriction is not necessary in the present considerations. Homogeneity is required only over a region of radius equal to ct since nothing further away can affect the cosmology or the elementary particle reactions to be discussed. At the universal age of ~ 600 seconds corresponding to the end of the period of this study, the nuclear mass enclosed in the sphere of influence is $\sim 10^{34}$ g, that is, ~ 5 solar masses, and is much less at earlier epochs. Another way of looking at this result is that lengths greater than ct, in particular R_0 and any gradient of R_0, must be negligible because of the finite velocity of propagation of disturbances.

[5] R. C. Tolman, *Relativity, Thermodynamics and Cosmology* (Clarendon Press, Oxford, 1934).

[6] A very small value of Λ may be used to adjust the present age of this model, although it is of no consequence during the early epochs of interest here [see G. Gamow, Revs. Modern Phys. **21**, 367 (1949)]. In this connection it may be of interest to note that while Eqs. (1) and (2) contain a density singularity at zero time, they also implicitly contain the conclusion that the duration or age of the expansion from this singularity is finite. Taking $\Lambda=0$ and neglecting terms containing $1/R_0^2$ for early epochs, one can show that this age is given in cgs units by the following integral:

$$\mathbf{a}=\int_0^{\mathbf{a}} dt=\left(\frac{3}{8\pi G}\right)^{\frac{1}{2}}\int_{\rho(\mathbf{a})}^{\infty}\frac{c^2d\rho}{\rho^{\frac{1}{2}}(p+\rho c^2)},$$

where p and ρ are total pressure and density. Since the pressure is positive,

$$\mathbf{a}\leqq\left(\frac{3}{8\pi G}\right)^{\frac{1}{2}}\int_{\rho(\mathbf{a})}^{\infty}\frac{d\rho}{\rho^{\frac{3}{2}}}=\left(\frac{3}{2\pi G\rho}\right)^{\frac{1}{2}}.$$

A lower bound on the duration may be obtained by noting that for a relativistic fluid $0\leqq p\leqq \rho c^2/3$. Hence

$$\mathbf{a}\geqq\left(\frac{3}{8\pi G}\right)^{\frac{1}{2}}\int_{\rho(\mathbf{a})}^{\infty}\frac{3d\rho}{4\rho^{\frac{3}{2}}}=\left(\frac{8}{3\pi G\rho}\right)^{\frac{1}{2}},$$

so that **a** is bounded.

As already mentioned, the cosmological model outlined above was a sufficient approximation in previous calculations of the neutron-capture theory[1] because the temperature taken for the start of element formation was well below the electron rest mass equivalent and, therefore, reactions among elementary particles and photons could be ignored. One has only to consider that the nucleons and nuclei formed remained in thermal equilibrium with the expanding radiation field. These previous calculations, which were based on the time scale described by Eq. (6), continue to be valid provided ρ_0 is adjusted as required to fit the time scale to be described in this paper. The adjustment required is insignificant.

The study of the induced and inverse β processes involving neutrons and protons prior to any appreciable element formation concerns much earlier epochs and therefore much higher temperatures. In this case one must consider positrons, electrons, neutrinos, antineutrinos (if distinguishable from neutrinos), and radiation. The equation of state for radiation only, implicit in Eqs. (4)–(6), may no longer be an adequate approximation. We shall suppose that this mixture of elementary particles and photons is at a sufficiently high temperature for equilibrium to be maintained, but we shall not require temperatures so high as to require nucleon pairs. Furthermore, the nucleons present are assumed to have a negligible effect on pressure, density, and temperature, since even for temperatures as low as $\sim$0.1 Mev the nucleon density is many orders of magnitude less than the radiation density.

The density and pressure of the constituents of the medium may be obtained from the Fermi-Dirac and Bose-Einstein distribution laws for the number of particles in the energy range dE at E, *viz.*,

$$N(E)dE=\frac{4\pi}{h^3}\sum{}'|\mathbf{p}|E[\exp(E/kT)\pm1]^{-1}dE, \quad (8)$$

where $|\mathbf{p}|$ is the momentum, and the summation, $\sum{}'$, is over charge and spin states. In the present calculation the number of particles and photons is not conserved so that a degeneracy parameter is not required. The density and pressure, according to Eq. (8), are given by

$$\rho(T)=\frac{4\pi}{c^2h^3}\sum{}'\int_{mc^2}^{\infty}|\mathbf{p}|E^2[\exp(E/kT)\pm1]^{-1}dE, \quad (9a)$$

and

$$p(T)=\frac{4\pi}{3h^3}\sum{}'\int_{mc^2}^{\infty}|\mathbf{p}|^3[\exp(E/kT)\pm1]^{-1}dE, \quad (9b)$$

where

$$|\mathbf{p}|=(1/c)(E^2-m^2c^4)^{\frac{1}{2}}. \quad (9c)$$

In particular, for electrons and positrons one obtains, with the transformation $E=m_ec^2\cosh\theta$, the following:

$$\rho_{e^-}=\rho_{e^+}=\frac{a_e}{c^2}\int_0^{\infty}\frac{\sinh^2\theta\cosh^2\theta d\theta}{1+\exp(x\cosh\theta)}\ \mathrm{g/cm^3}, \quad (10a)$$

$$p_{e^-}=p_{e^+}=\frac{a_e}{3}\int_0^{\infty}\frac{\sinh^4\theta d\theta}{1+\exp(x\cosh\theta)}\ \mathrm{dynes/cm^2}. \quad (10b)$$

In these equations

$$a_e=8\pi m_e^4c^5/h^3; \quad (11a)$$

m_e and h are the electron rest mass and Planck's constant, respectively;

$$x=m_ec^2/(kT) \quad (11b)$$

defines temperature in units of the electron rest mass, and k is Boltzmann's constant. Spin states have been counted in the above expressions, and the total electron energy includes rest mass.

In order to carry out numerical calculations, it should be noted that the definite integrals in the expressions for $p_e(x)$ and $\rho_e(x)$ can be expanded in series of modified Bessel functions $K_i(nx)$. One can write

$$f_0=\int_0^{\infty}[1+\exp(x\cosh\theta)]^{-1}d\theta=\sum_{n=1}^{\infty}(-1)^{n+1}K_0(nx), \quad (12)$$

$$f_1=\int_0^{\infty}\sinh^2\theta[1+\exp(x\cosh\theta)]^{-1}d\theta=x^{-1}\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1}K_1(nx), \quad (13)$$

and

$$f_2=\int_0^{\infty}\sinh^4\theta[1+\exp(x\cosh\theta)]^{-1}d\theta=3x^{-2}\sum_{n=1}^{\infty}(-1)^{n+1}n^{-2}K_2(nx), \quad (14)$$

so that

$$\rho_e(x)=\rho_{e^-}+\rho_{e^+}=(2a_e/c^2)(f_1+f_2), \quad (15)$$

and

$$p_e(x)=p_{e^-}+p_{e^+}=(2a_e/3)f_2. \quad (16)$$

In the high temperature limit, $kT\gg mc^2$, which is equivalent to setting $m=0$ in Eqs. (9), the density and pressure for all Bose-Einstein particles approach those for photons except for factors which depend on spin and charge states. Similarly, for Fermi-Dirac particles the density and pressure approach those for neutrinos, again except for a factor which accounts for differing charge and spin states.

For radiation, taking into account the two states of polarization, one obtains the following from the Bose-Einstein integral:

$$\rho_\gamma=\frac{a_\gamma}{c^2}T^4=\left(\frac{\pi^4a_e}{15c^2}\right)x^{-4}\ \mathrm{g/cm^3}, \quad (17)$$

and

$$p_\gamma = \tfrac{1}{3}\rho_\gamma c^2 \text{ dynes/cm}^2, \tag{18}$$

where

$$a_\gamma = 8\pi^5 k^4/15c^3h^3. \tag{18a}$$

For neutrinos we consider two cases,[7] namely, neutrinos and antineutrinos indistinguishable ($\nu \equiv \nu^*$) and distinguishable ($\nu \not\equiv \nu^*$). For the temperature range in which the neutrinos are in thermal equilibrium with the other constitutents of the medium, the Fermi-Dirac integral gives

for $\nu \equiv \nu^*$:

$$\rho_\nu = \tfrac{7}{8}\rho_\gamma, \tag{19}$$

and

$$p_\nu = \tfrac{7}{8}p_\gamma; \tag{20}$$

for $\nu \not\equiv \nu^*$:

$$\rho_\nu = \rho_{\nu^*} = \tfrac{7}{8}\rho_\gamma, \tag{21}$$

and

$$p_\nu = p_{\nu^*} = \tfrac{7}{8}p_\gamma, \tag{22}$$

so that the neutrino pressure and density in the latter case are twice those in the former. It should be noted that the results stated in Eqs. (19)–(22) are predicated on the assumption that no type of particle is degenerate in the present problem. The simple expressions for the neutrino density and pressure given in Eqs. (19)–(22) hold for all Fermi-Dirac particles in the limit of sufficiently high temperature, i.e., there is a contribution to the density of $(7/16)\rho_\gamma$ for each degree of freedom. Similarly for Bose-Einstein particles there is a contribution to the density of $\frac{1}{2}\rho_\gamma$ for each degree of freedom.

It can be shown from Eq. (9) that for a Fermi-Dirac particle of mass, m_i,

$$\rho_i(x) \propto \left(\frac{m_i}{m_e}\right)^4 \rho_e[(m_i/m_e)x],$$

with the proportionality factor depending on the previously mentioned spin and charge states. Thus all Fermi-Dirac particles exhibit the same behavior provided that an appropriate shift is made in the temperature scale. A similar result can be obtained for Bose-Einstein particles. The qualitative behavior of ρ_i *versus* T after suitable normalization of the temperature scales is essentially the same for fermions and bosons.

The neutrino contribution given by Eqs. (19)–(22) to the total pressure and density requires modification for the temperature range of interest in calculating the proton-neutron ratio as a function of the time. At very high temperatures the neutrino component maintains itself in equilibrium with the other constituents of of the medium through interaction with mesons. When the medium has expanded and cooled somewhat below a temperature equivalent to the rest mass of the lightest meson, the neutrinos freeze in and continue to expand and cool adiabatically as would a pure radiation gas. After this freeze-in the neutrino temperature will differ from that of the other components of the medium. It will be seen that the freeze-in must have occurred at a temperature higher than is required for neutrons and protons to be very nearly in thermodynamic equilibrium. For the temperature region of interest, then, we must deal with nucleons, electrons, positrons, and radiation at one temperature, and neutrinos at another temperature. The calculation of the neutron-proton ratio does not require that a specific freeze-in temperature be given, but only that neutrinos be frozen in before an appreciable fraction of the electron pairs start to annihilate.

It is of some interest to examine in more detail the freezing in of neutrinos during the period from $\sim$15 to $\sim$5 Mev. Non-equilibrium reactions involving neutrinos become important only below $\sim$5 Mev. When the temperature was well above the rest mass equivalent of mesons, the neutrinos maintained equilibrium through interaction with mesons. At such temperatures the contribution of mesons to the density was $3.25\rho_\gamma$, while the total contribution due to photons, electrons, positrons, and neutrinos was $3.625\rho_\gamma$ or $4.50\rho_\gamma$, for $\nu \equiv \nu^*$ and $\nu \not\equiv \nu^*$, respectively.[8] Since the meson rest energy is distributed uniformly among the lighter particles when the mesons annihilate, it is clear that the number of neutrinos will about double when meson annihilation occurs. Now the bulk of mesons will annihilate when the temperature in the universal expansion has dropped significantly below that equivalent to $m_\mu c^2$ ($\sim$108 Mev) or $m_\pi c^2$ ($\sim$138 Mev), down to 10 Mev. At 10 Mev the Boltzmann factors for μ and π mesons are $\sim 2\times 10^{-5}$ and $\sim 10^{-6}$, respectively. This temperature decrease, as will be seen later when the time scale for the cosmological model is calculated, requires a duration of $\sim 10^{-2}$ sec in the universal expansion.

The meson reactions $\pi^\pm \rightleftharpoons \mu^\pm + \nu$ and $\mu^\pm \rightleftharpoons e^\pm + 2\nu$ are very fast, even if one neglects induced decay, having lifetimes of $\sim 2\times 10^{-8}$ sec and $\sim 2\times 10^{-6}$ sec, respectively. Since the concentrations of neutrinos and mesons are comparable, the reaction rate $1/(2\times 10^{-8})$ per second per neutrino is $\sim 10^6$ times the equilibrium rate (due to annihilation) of $\sim 1/10^{-2}$ per second per neutrino. Hence between $\sim$100- and $\sim$10-Mev thermal equilibrium holds. By 5 Mev, however, the Boltzmann factor $\exp(-m_\mu c^2/kT) \cong \exp(-138/5)$ has reduced the reaction rate to insignificance even though there is a

[7] Recently, theoretical arguments in favor of distinguishability, i.e., against the Majorana theory of neutral particles, have been given by E. R. Caianiello, Phys. Rev. **86**, 564 (1952). However, we consider both cases throughout this paper because it does not appear to be a settled question at this time. [See also C. S. Wu, Physica **18**, 989 (1952).]

[8] As has been shown, in the high temperature limit the Fermi-Dirac μ^+ and μ^- mesons each contribute $(7/8)\rho_\gamma$, while the Bose-Einstein π^+, π^-, and π^0 mesons each contribute $(1/2)\rho_\gamma$ for a total of $3.25\rho_\gamma$. Electrons and positrons each contribute $(7/8)\rho_\gamma$, neutrinos contribute $(7/8)\rho_\gamma$ or $2(7/8)\rho_\gamma$ according as $\nu \equiv \nu^*$ or $\nu \not\equiv \nu^*$, and photons contribute ρ_γ for a total of $3.625\rho_\gamma$ or $4.50\rho_\gamma$. The numerical factors obtained here depend on the discussion following Eq. (17).

good deal more time available for reactions to take place due to the reduced rate of cooling in the universal expansion. Hence the residual mesons cannot transfer a significant amount of rest mass energy to the neutrino gas, although almost all the meson rest mass energy is uniformly distributed.

Having described the nature of the medium, we can now proceed to determine the universal expansion rate for the period of interest in this problem. The rate of expansion for early times, Eq. (3), can be written in cgs units as

$$\frac{1}{2}\frac{dg}{dt}=\frac{1}{l}\frac{dl}{dt}=\frac{d\ln l}{dt}=\left(\frac{8\pi G}{3}\right)^{\frac{1}{2}}\rho^{\frac{1}{2}}, \tag{23}$$

where ρ, the total density, may now be written

$$\rho(x,l)=\rho(x)+\rho(l), \tag{24}$$

for the following reason. The quantity

$$\rho(x)=\rho_{e^+}+\rho_{e^-}+\rho_\gamma \tag{24a}$$

depends only on the temperature, while

$$\rho(l)=\rho_\nu\ (\text{or } \rho_\nu+\rho_{\nu^*}) \tag{24b}$$

depends only on the proper distance l, since the neutrinos are expanding adiabatically as a radiation gas after freeze-in. We can, in fact, write $\rho_\nu\propto l^{-4}$, so that Eq. (23) can be rewritten as follows, after differentiation with respect to time:

$$\frac{d^2g}{dt^2}=\left(\frac{8\pi G}{3\rho}\right)^{\frac{1}{2}}\left(\frac{\partial\rho}{\partial\ln x}\frac{d\ln x}{dt}+\frac{\partial\rho}{\partial\ln l}\frac{d\ln l}{dt}\right). \tag{25}$$

Substituting for $\rho^{\frac{1}{2}}$ from Eq. (23) and using Eq. (24) yields

$$\frac{d^2g}{dt^2}=\frac{8\pi G}{3}\left[\frac{d\rho(x)}{d\ln x}\frac{d\ln x}{d\ln l}-4\rho(l)\right]. \tag{26}$$

If now we add Eqs. (1a) and (1b), neglect terms containing $1/R_0^2$, and convert to cgs units, we obtain

$$\frac{d^2g}{dt^2}=-\left(\frac{8\pi G}{c^2}\right)(p+\rho c^2)$$
$$=-\left(\frac{8\pi G}{c^2}\right)[p(x)+p(l)+\rho(x)c^2+\rho(l)c^2]. \tag{27}$$

If we equate Eqs. (26) and (27) and note that $(4/3)\rho(l)c^2=p(l)+\rho(l)c^2$, then we obtain

$$\frac{d\ln l}{d\ln x}=\frac{-c^2}{3[p(x)+\rho(x)c^2]}\frac{d\rho(x)}{d\ln x}, \tag{28}$$

where

$$\frac{d\rho(x)}{d\ln x}=\frac{d\rho_e(x)}{d\ln x}-\frac{3}{c^2}[p_\gamma(x)+\rho_\gamma(x)c^2]. \tag{28a}$$

This result is independent of the presence of the frozen-in neutrinos. Equation (28) can now be integrated in the following manner. From Eqs. (10) one obtains

$$\frac{dp_e(x)}{d\ln x}=-(p_e+\rho_e c^2), \tag{29}$$

or, more conveniently,

$$c^2x^4\frac{d\rho_e}{d\ln x}=-3x^4(p_e+\rho_ec^2)+\frac{d}{d\ln x}[x^4(p_e+\rho_ec^2)]. \tag{29a}$$

Employing Eq. (29a) in Eqs. (28) yields the desired result, namely,

$$\frac{d\ln l}{d\ln x}=1-\frac{\frac{d}{d\ln x}\{x^4[p(x)+\rho(x)c^2]\}}{3x^4[p(x)+\rho(x)c^2]}. \tag{30}$$

Since the adiabatic expansion of a radiation universe leads[1] to $d\ln l/d\ln x=1$, the second term in Eq. (30) represents a correction to the description of the cosmological model previously used with the neutron-capture theory, a correction which accounts for the interconversion of matter and radiation. Equation (30) may be integrated to yield

$$\ln l=\ln x-\tfrac{1}{3}\ln\{x^4[p(x)+\rho(x)c^2]\}+\text{constant}. \tag{31}$$

Finally, using Eqs. (10)–(14), one can write Eq. (25) as

$$\frac{d\ln l}{d\ln x}=1+\frac{2a_e(f_0+2f_1)}{3[p(x)+\rho(x)c^2]}. \tag{32}$$

As will become evident, Eq. (32) is required in order to obtain the explicit time dependence of the temperature.

The neutrino temperature T_ν (or $x_\nu=m_ec^2/kT_\nu$) may be determined from Eq. (30) by recalling that during the period of interest the neutrinos expand and cool adiabatically, so that $x_\nu=f(l)$ only, and in fact, $x_\nu\propto l$. Then it follows that Eq. (31) can be written as

$$\ln x_\nu=\ln x-\tfrac{1}{3}\ln\{x^4[p(x)+\rho(x)c^2]\}+\text{constant}. \tag{33}$$

The constant of integration can be evaluated by noting that for small x (high temperatures) the neutrino temperature approaches the temperature of the rest of the medium. In fact, as $x\rightarrow0$, $x_\nu\rightarrow x$ so that the integration constant becomes

$$\text{constant}=\tfrac{1}{3}\ln\{x^4[p(x)+\rho(x)c^2]\}\,|_{x=0}. \tag{34}$$

From the definition of p_γ and ρ_γ it is evident [see discussion following Eq. (22)] that, for any x, $\rho_\gamma c^2x^4=\text{constant}=\pi^4a_e/15$, and $p_\gamma x^4=\tfrac{1}{3}\rho_\gamma c^2x^4$. Since

$$\lim_{x\rightarrow0}p_ex^4=2(7/8)p_\gamma x^4,\quad \lim_{x\rightarrow0}\rho_ec^2x^4=2(7/8)\rho_\gamma c^2x^4,$$

TABLE I. Time scales, rate coefficients, and quantum statistical integrals.[a]

T (°K)	x	t (sec) $\nu\equiv\nu^*$	t (sec) $\nu\not\equiv\nu^*$	t (sec) rad. model[b]	$\frac{15x^4}{\pi^4}f_0$	$\frac{15x^4}{\pi^4}f_1$	$\frac{15x^4}{\pi^4}f_2$	K_n	K_p
∞	0	0	0	0	0	0	0.87500	∞	∞
5.930×10^{10}	0.1	0.04	0.04	0.06	2.2105×10^{-5}	1.2535×10^{-3}	0.87311	5.07×10^{6}	3.94×10^{6}
2.965×10^{10}	0.2	0.14	0.12	0.26	2.6866×10^{-4}	4.9011×10^{-3}	0.86754	1.91×10^{5}	1.12×10^{5}
1.482×10^{10}	0.4	0.55	0.50	1.05	2.9583×10^{-3}	1.8300×10^{-2}	0.84626	7.58×10^{3}	2.51×10^{3}
9.884×10^{9}	0.6	1.27	1.14	2.36	1.1131×10^{-2}	3.7625×10^{-2}	0.81339	1.17×10^{3}	2.34×10^{2}
7.413×10^{9}	0.8	2.26	2.02	4.20	2.6998×10^{-2}	6.0185×10^{-2}	0.77190	3.09×10^{2}	37.8
5.930×10^{9}	1.0	3.58	3.21	6.57	5.1303×10^{-2}	8.3590×10^{-2}	0.72406	1.18×10^{2}	9.37
2.965×10^{9}	2.0	15.65	14.12	26.28	0.25584	0.16512	0.46121	6.88	4.00×10^{-2}
1.977×10^{9}	3.0	39.72	36.05	59.11	0.41847	0.16423	0.25402	2.38	9.49×10^{-4}
1.482×10^{9}	4.0	78.99	72.04	105.20	0.43426	0.12227	0.12828	1.91	2.69×10^{-5}
1.186×10^{9}	5.0	134.15	122.64	164.25	0.35354	7.7674×10^{-2}	6.1253×10^{-2}	1.74	···
9.884×10^{8}	6.0	203.84	186.56	236.49	0.24783	4.4664×10^{-2}	2.8128×10^{-2}	···	···
8.472×10^{8}	7.0	285.68	261.76	321.90	0.15696	2.3982×10^{-2}	1.2552×10^{-2}	···	···
7.413×10^{8}	8.0	378.40	347.03	420.44	9.2363×10^{-2}	1.2248×10^{-2}	5.4786×10^{-3}	···	···
6.589×10^{8}	9.0	481.59	442.06	532.17	5.1402×10^{-2}	6.0209×10^{-3}	2.3499×10^{-3}	···	···
5.930×10^{8}	10.0	595.56	546.96	657.02	2.7379×10^{-2}	2.8716×10^{-3}	9.9368×10^{-4}	···	···
0	∞	∞	∞	∞	0	0	0	1.63	0

[a] The universal constants employed in these calculations are those given by J. A. Bearden and H. M. Watts, Phys. Rev. **81**, 73 (1951). Note that the limiting values at high temperatures do not include any contributions from mesons.
[b] This column gives the time scale for the pure radiation model described by Eqs. (4)–(7).

it follows that Eq. (33) can be written as

$$\left(\frac{x_\nu}{x}\right)^3=\left(\frac{T}{T_\nu}\right)^3=\frac{2.75(p_\gamma+\rho_\gamma c^2)}{p(x)+\rho(x)c^2}, \tag{35}$$

from which the neutrino temperature can be determined for any value of x. For the sake of completeness it should be noted that

$$\lim_{x\to\infty} p_e x^4=0,\quad \lim_{x\to\infty}\rho_e c^2x^4=0,$$

while the quantities $p_\gamma x^4$ and $\rho_\gamma c^2 x^4$ are constants for all x, as just described.

One other relationship which we shall require is that between temperature and time. This is obtained by multiplying Eq. (23) by $d\ln x/d\ln l$, as evaluated from Eq. (32), with the following result:

$$\frac{d\ln x}{dt}=\frac{d\ln x}{d\ln l}\frac{d\ln l}{dt}=\left(\frac{8\pi G}{3}\rho\right)^{\frac{1}{2}}\frac{d\ln x}{d\ln l}. \tag{36}$$

The integration of Eq. (36) (performed to an accuracy better than 0.1 percent on a Maddida, a digital differential analyzer built by Northrup Aircraft, Inc.) for the two cases $\nu\equiv\nu^*$ and $\nu\not\equiv\nu^*$ gives the time in the universal expansion as a function of x, and these quantities are given in Table I. For comparison, Table I also contains the time as a function of x for the expanding cosmological model containing radiation only [see Eq. (6)]. Other quantities given in Table I are the series of modified Bessel functions f_0, f_1, and f_2, as defined in Eqs. (12)–(14), which are used in computing pressure and density. In Table II are given the total density ρ

TABLE II. Neutrino temperature; total, radiation, and electron-pair densities; and universal expansion rates.[a]

T (°K)	x	x_ν/x	ρ_γ (g/cm³)	ρ/ρ_γ ($\nu\equiv\nu^*$)	ρ/ρ_γ ($\nu\not\equiv\nu^*$)	ρ_e/ρ_γ	$\frac{d\ln l}{d\ln x}$	$\frac{d\ln l}{dt}$ (sec⁻¹) ($\nu\equiv\nu^*$)	$\frac{d\ln l}{dt}$ (sec⁻¹) ($\nu\not\equiv\nu^*$)
∞	0	1.0000	∞	3.625	4.500	1.750	1.0000	∞	∞
5.930×10^{10}	0.1	1.0002	7.226×10^{7}	3.623	4.497	1.749	1.0005	14.51	16.17
2.965×10^{10}	0.2	1.0009	4.516×10^{6}	3.617	4.488	1.745	1.0018	3.625	4.039
1.482×10^{10}	0.4	1.0037	2.822×10^{5}	3.591	4.454	1.729	1.0073	0.9031	1.006
9.884×10^{9}	0.6	1.0082	5.575×10^{4}	3.549	4.396	1.702	1.0161	0.3990	0.4441
7.413×10^{9}	0.8	1.0145	1.764×10^{4}	3.490	4.317	1.664	1.0280	0.2226	0.2475
5.930×10^{9}	1.0	1.0224	7.226×10^{3}	3.416	4.217	1.615	1.0424	0.1409	0.1566
2.965×10^{9}	2.0	1.0821	4.516×10^{2}	2.891	3.529	1.253	1.1350	3.241×10^{-2}	3.581×10^{-2}
1.977×10^{9}	3.0	1.1616	89.20	2.317	2.798	0.836	1.2129	1.290×10^{-2}	1.417×10^{-2}
1.482×10^{9}	4.0	1.2407	28.22	1.870	2.240	0.501	1.2357	6.517×10^{-3}	7.132×10^{-3}
1.186×10^{9}	5.0	1.3044	11.56	1.580	1.882	0.278	1.2054	3.834×10^{-3}	4.184×10^{-3}
9.884×10^{8}	6.0	1.3478	5.575	1.411	1.676	0.146	1.1501	2.516×10^{-3}	2.742×10^{-3}
8.472×10^{8}	7.0	1.3736	3.009	1.319	1.564	0.073	1.0966	1.787×10^{-3}	1.946×10^{-3}
7.413×10^{8}	8.0	1.3876	1.764	1.271	1.508	0.035	1.0568	1.343×10^{-3}	1.463×10^{-3}
6.589×10^{8}	9.0	1.3947	1.101	1.248	1.479	0.017	1.0313	1.052×10^{-3}	1.145×10^{-3}
5.930×10^{8}	10.0	1.3981	0.7226	1.237	1.466	0.008	1.0165	8.480×10^{-4}	9.231×10^{-4}
0	∞	1.4010	0	1.227	1.454	0	1.0000	0	0

[a] The universal constants employed in these calculations are those given by J. A. Bearden and H. M. Watts, Phys. Rev. **81**, 73 (1951). Note that the limiting values at high temperatures do not include any contributions from mesons.

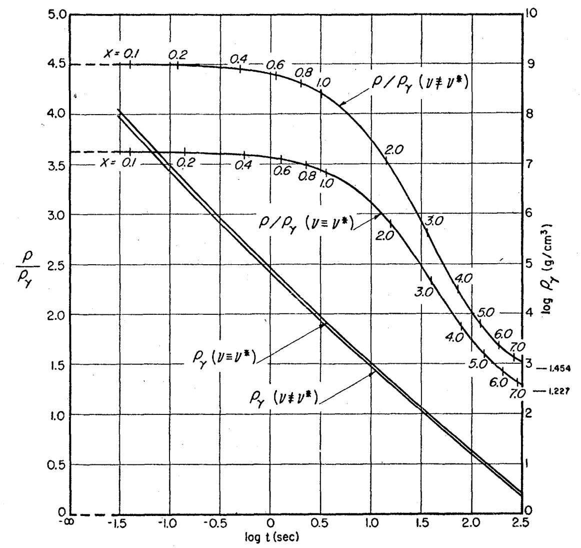

FIG. 1. Total density ρ, in units of photon density ρ_γ, and ρ_γ *versus* time during the very early epochs of the expanding universe for Majorana and Dirac neutrinos. The corresponding temperatures are given in terms of $x=m_ec^2/kT$. The ρ/ρ_γ curves are extrapolated to $t=0$ without regard to the presence of mesons and other elementary particles.

(i.e., the density of electrons, positrons, neutrinos, and radiation) and the density of electrons plus positrons, ρ_e, in units of the radiation density, ρ_γ, the neutrino temperature expressed as $x_\nu/x=T/T_\nu$, the differential quotient $d\ln l/d\ln x$, and finally the expansion rate $d\ln l/dt$.

Several interesting features of this cosmological model are evident upon examination of Tables I and II. First, the temperature drops much more rapidly in the nonstatic model with interconversion of matter and radiation than it does in the model of adiabatically expanding radiation only.[9] However the cases of distinguishable and indistinguishable neutrinos differ very little in this respect. The total density ρ does not drop off very greatly until the universe has cooled to about the electron rest mass equivalent. At this point the large density contribution of electron pairs begins to decrease sharply [see Table II] as the pairs disappear by annihilation into the radiation field which has fewer degrees of freedom. This behavior is demonstrated in Fig. 1. Next, if one recalls that the expanding model of radiation only is represented by $d\ln l/d\ln x=1$, then one can see in Table II that the maximum deviation from this model due to the interconversion of matter and radiation is $\sim$24 percent, and this occurs at 0.125 Mev ($\sim m_ec^2/4$). This deviation represents a more rapid expansion rate than in the pure radiation model and, in fact, the expansion rate $d\ln l/dt$ is higher in the model with interconversion by just the factor $(\rho/\rho_\gamma)^{\frac{1}{2}}$. Finally, it should be noted that x_ν/x (a quantity which does not depend on whether $\nu\equiv\nu^*$ or $\nu\not\equiv\nu^*$) differs from unity by less than one percent until x has increased to about 0.7 or $kT\cong 0.73$ Mev. At $x=0.1$, where $kT=5$ Mev, the deviation is 0.02 percent. It is then quite clear that selecting say 5–10 Mev as the freeze-in temperature for neutrinos is not only reasonable but quite an adequate approximation. It should be noted that the mathematical limits approached as $x\rightarrow\infty$ for all the quantities given in Tables I and II are included for the sake of completeness. However, the behavior of the cosmological model discussed changes at longer times, $x\sim 10^7$, when the density of matter exceeds the density of radiation.

In the next section we shall calculate the relative concentrations of neutrons and protons as a function of the time in the universal expansion. This ratio, it will be recalled, plays a most important role in determining the relative abundances of the nuclear species as calculated according to the simple neutron-capture theory of element formation including the effects of thermonuclear reactions. As has been stated, this theory quite clearly requires that during the early epochs in the universal expansion the density of nucleons, and of the nuclear species formed, should be negligibly small compared with the radiation density. Consequently, the physical conditions in the expanding model as described in Tables I and II, in which nucleon density is taken to be negligible, are used as a basis for examining the various non-equilibrium reactions between neutrons and protons.

III. THE NEUTRON-PROTON RATIO

In this section we shall examine the reactions which may occur among neutrinos, electrons, positrons, and nucleons in the very early stages of the cosmological model described in the previous section. In particular we shall calculate the ratio of the concentrations of protons and neutrons as a function of time, a ratio upon which the results of the neutron-capture theory of element formation strongly depend.

The nuclear reactions which must be considered in determining the proton-neutron ratio are the following:[10]

$$n+e^+\rightleftharpoons p+\nu, \tag{37a}$$

$$n+\nu\rightleftharpoons p+e^-, \tag{37b}$$

$$n\rightleftharpoons p+e^-+\nu. \tag{37c}$$

[9] There are perhaps slight shifts in the expansion time scale, much too small to appear with the number of significant figures given in Tables I and II. These are caused by the presence of and annilation of mesons, nucleons, gravitons, etc., between $x=0$ and $x=0.1$, should such particles exist during this epoch, and should the relativistic cosmology apply at the extreme conditions existing during this very brief early period. There is, however, serious doubt that the cosmology applies and, since we are interested only in the epochs of temperature lower than $x=0.1$, we can for the present perhaps ignore this problem and accept the insignificant additive constant in the time scale. See reference 1 as well as A. Einstein, *The Meaning of Relativity* (Princeton University Press, Princeton, 1945).

[10] The question of the existence of the antineutrino is of no concern in determining the individual reaction rates (see reference 7). In the absence of charge, mass, and magnetic moment, the absorption of a neutrino from a negative energy state is in all respects equivalent to the emission of an antineutrino.

The probability per second w for these reactions may be obtained from the Fermi theory of β decay. For the reaction $n+e^+\rightarrow p+\nu$, one has, per electron

$$w_{A'}=\frac{2\pi}{\hbar}(|\psi_p(0)|\cdot|\psi_\nu(0)|\cdot|\mathfrak{M}|g)^2\frac{dn}{dE}\ \mathrm{sec}^{-1}, \quad (38)$$

where the expectation value at the origin for the product particles, proton, and neutrino, depends on $|\psi_p(0)|^2|\psi_\nu(0)|^2$, the ψ are taken as plane wave states, $\mathfrak{M}$, the matrix element, is taken as unity for lack of a better estimate, g is the Fermi coupling constant and the quantity dn/dE is the energy density of available final states. We can, therefore, write

$$w_{A'}=\frac{2\pi g^2}{\hbar\Omega^2}\frac{dn}{dE}\ \mathrm{sec}^{-1}, \quad (39)$$

where Ω is any finite normalization volume. For the neutrino the number of available states per unit energy in the volume Ω is the difference between the total and occupied number of states, *viz.*,

$$\frac{4\pi(2i_\nu+1)|\mathbf{p}_\nu|}{2\pi\hbar^3}\{1-[1+\exp(E_\nu/kT_\nu)]^{-1}\}\frac{d|\mathbf{p}_\nu|}{dE_\nu}, \quad (40a)$$

while for each neutron the number of available final states in this particular reaction is $(2i_p+1)$. Since all neutrons are equivalent, one may write for the total number of final states in Ω:

$$\Omega n_n(2i_p+1)=2\Omega n_n. \quad (40b)$$

In Eqs. (40) the neutrino and proton spins are denoted by i_ν and i_p, $|\mathbf{p}_\nu|$ is the neutrino momentum and n_n the number of neutrons per unit volume. Since dn/dE n Eq. (39) is the product of terms given by Eq. (40a) and Eq. (40b) one can write

$$w_{A'}=\frac{4g^2n_n|\mathbf{p}_\nu|\exp(E_\nu/kT_\nu)}{\pi\hbar^4[1+\exp(E_\nu/kT_\nu)]}\frac{d|\mathbf{p}_\nu|}{dE_\nu}\ \mathrm{sec}^{-1}. \quad (41)$$

The number of such reactions, $n+e^+\rightarrow p+\nu$, per second per unit volume, is given by

$$A'n_nn_{e^+}=\int_{m_ec^2}^{\infty}w_{A'}n_{e^+}(E_{e^+})dE_{e^+}, \quad (42)$$

where

$$E_\nu=E_{e^+}+Q; \quad (42a)$$

$$Q=(m_n-m_p)c^2 \quad (42b)$$

is the neutron-proton energy difference and n_{e^+} is the concentration of positrons per unit energy at E_{e^+}. The lower limit of integration is the threshold energy, which in this case is the electron rest energy. Using the relation $E_\nu=|\mathbf{p}_\nu|c$ and replacing n_{e^+} by means of the Fermi-Dirac integral, *viz.*,

$$n_{e^+}(|\mathbf{p}_{e^+}|)d|\mathbf{p}_{e^+}|=\frac{8\pi|\mathbf{p}_{e^+}|^2d|\mathbf{p}_{e^+}|}{(2\pi\hbar^3)[1+\exp(E_{e^+}/kT)]}, \quad (43)$$

where $|\mathbf{p}_{e^+}|$ is given by Eq. (9c), one can write Eq. (42) in the form

$$A'n_nn_{e^+}=a_0n_n\int_{m_ec^2}^{\infty}\frac{E_{e^+}(E_{e^+}+Q)^2(E_{e^+}^2-m_e^2c^4)^{\frac{1}{2}}\exp[(E_{e^+}+Q)/kT_\nu]}{\{1+\exp[(E_{e^+}+Q)/kT_\nu]\}\{1+\exp[E_{e^+}/kT]\}}dE_{e^+}, \quad (44)$$

where Eq. (42a) has been used to eliminate E_ν, and

$$a_0=(4g^2)/(\pi^3c^6\hbar^7). \quad (44a)$$

It should be recalled that the neutrino temperature $T_\nu\neq T$, where the latter is the temperature of the remainder of the medium. Equation (44) may be rewritten by taking

$$x=m_ec^2/(kT),\quad x_\nu=m_ec^2/(kT_\nu),\quad q=Q/(m_ec^2),\quad \epsilon=E_e/(m_ec^2), \quad (45)$$

with the result that for the reaction $n+e^+\rightarrow p+\nu$,

$$A'n_nn_{e^+}=m_e^5c^{10}a_0n_nI_{A'}\ \mathrm{sec}^{-1}\ \mathrm{cm}^{-3}, \quad (46)$$

where

$$I_{A'}=\int_1^\infty\frac{\epsilon(\epsilon+q)^2(\epsilon^2-1)^{\frac{1}{2}}\exp[(\epsilon+q)x_\nu]d\epsilon}{\{1+\exp[\epsilon x]\}\{1+\exp[(\epsilon+q)x_\nu]\}}. \quad (46a)$$

Rates for the other reactions in Eq. (37a) and Eq. (37b) may be obtained from similar considerations, since they are all of second degree. One obtains the following:

for $p+\nu\rightarrow n+e^+$,

$$An_pn_\nu=m_e^5c^{10}a_0n_pI_A\ \mathrm{sec}^{-1}\ \mathrm{cm}^{-3}, \quad (47)$$

where

$$I_A=\int_1^\infty\frac{\epsilon(\epsilon+q)^2(\epsilon^2-1)^{\frac{1}{2}}\exp(\epsilon x)d\epsilon}{\{1+\exp[\epsilon x]\}\{1+\exp[(\epsilon+q)x_\nu]\}}, \quad (47a)$$

and where

$$E_{e^+}=E_\nu-Q; \quad (47b)$$

for $n+\nu\rightarrow p+e^+$,

$$B'n_nn_\nu=m_e^5c^{10}a_0n_nI_{B'}\ \mathrm{sec}^{-1}\ \mathrm{cm}^{-3}, \quad (48)$$

where

$$I_{B'}=\int_q^\infty\frac{\epsilon(\epsilon-q)^2(\epsilon^2-1)^{\frac{1}{2}}\exp(\epsilon x)d\epsilon}{\{1+\exp[\epsilon x]\}\{1+\exp[(\epsilon-q)x_\nu]\}}, \quad (48a)$$

and where

$$E_\nu=E_{e^-}-Q; \quad (48b)$$

for $p+e^-\rightarrow n+\nu$,

$$Bn_pn_{e^-}=m_e^5c^{10}a_0n_pI_B\ \mathrm{sec}^{-1}\ \mathrm{cm}^{-3}, \quad (49)$$

where

$$I_B=\int_q^\infty \frac{\epsilon(\epsilon-q)^2(\epsilon^2-1)^{\frac{1}{2}}\exp[(\epsilon-q)x_\nu]d\epsilon}{\{1+\exp[\epsilon x]\}\{1+\exp[(\epsilon-q)x_\nu]\}}, \quad (49a)$$

and where

$$E_\nu=E_{e^-}-Q. \quad (49b)$$

The reaction rates for free neutron decay and the inverse process, Eq. (37c), require a slightly different calculation. Thus for the reaction $p+e^-+\nu\rightarrow n$, we note that the quantity dn/dE in Eq. (39) is given by just the number of protons present in the volume Ω, so that

$$w_C=(2\pi g^2/\hbar)n_p(2i_n+1),$$

and

$$Cn_pn_{e^-}n_\nu=\int_{m_ec^2}^Q w_C(E_{e^-})n_{e^-}(E_{e^-})n_\nu(Q-E_{e^-})dE_{e^-}, \quad (50)$$

where

$$E_{e^-}=Q-E_\nu. \quad (50a)$$

In Eq. (50) $n_{e^-}(E_{e^-})$ is the concentration of electrons per unit energy at E_{e^-} and $n_\nu(Q-E_{e^-})$ is the concentration of neutrinos per unit energy at $(Q-E_{e^-})$, the argument of n_ν being that required for energy balance in this reaction. Finally, one can rewrite Eq. (50) replacing n_{e^-} and n_ν by means of the Fermi-Dirac integral [see Eq. (43)] and using Eqs. (45), as

$$Cn_pn_{e^-}n_\nu=m_e^5c^{10}a_0n_pI_C \text{ sec}^{-1}\text{ cm}^{-3}, \quad (51)$$

where

$$I_C=\int_1^q \frac{\epsilon(\epsilon-q)^2(\epsilon^2-1)^{\frac{1}{2}}d\epsilon}{\{1+\exp[\epsilon x]\}\{1+\exp[(q-\epsilon)x_\nu]\}}. \quad (51a)$$

For the reaction $n\rightarrow p+e^-+\nu$, we note that dn/dE in Eq. (39) is the product of three quantities, *viz.*, the available states per unit energy in the volume Ω for protons, electrons, and neutrinos. For protons the number of available states is the number of neutrons in the volume Ω, *viz.*, $\Omega n_n(2i_n+1)$, while for electrons and neutrinos one can use the form of Eq. (40a) which gives this quantity for Fermi-Dirac particles. Since formally the reaction rate for free neutron decay is

$$C'n_n=\int_{m_ec^2}^Q w_{C'}dE, \quad (52)$$

it follows from Eq. (39) after some manipulation that

$$C'n_n=m_e^5c^{10}a_0n_nI_{C'} \text{ sec}^{-1}\text{ cm}^{-3}, \quad (53)$$

where

$$I_{C'}=\int_1^q \frac{\epsilon(\epsilon-q)^2(\epsilon^2-1)^{\frac{1}{2}}\exp[(q-\epsilon)x_\nu]\exp(\epsilon x)d\epsilon}{\{1+\exp[\epsilon x]\}\{1+\exp[(q-\epsilon)x_\nu]\}}. \quad (53a)$$

The foregoing reaction rates have been used in the equations developed below which describe the time dependence of neutron and proton concentrations. Let N_j be the number of nucleons of species j in the arbitrary finite volume V. The time derivative of N_j can be expressed formally for two- and three-body processes as

$$\frac{dN_j}{dt}=\sum_{\alpha,\beta}K_{\alpha\beta}n_\alpha N_\beta+\sum_{\alpha,\beta,\gamma}K_{\alpha\beta\gamma}n_\alpha n_\beta N_\gamma, \quad (54)$$

where

$$n_\alpha=N_\alpha/V. \quad (54a)$$

We note that

$$\frac{dn_j}{dt}=\frac{1}{V}\frac{dN_j}{dt}-\frac{n_j}{V}\frac{dV}{dt}, \quad (55)$$

where, since $V\propto l^3$,

$$\frac{1}{V}\frac{dV}{dt}=\frac{3}{l}\frac{dl}{dt}. \quad (55a)$$

For the cosmological model described in Sec. II,

$$\frac{1}{l}\frac{dl}{dt}=\left(\frac{8\pi G}{3}\rho\right)^{\frac{1}{2}},$$

where ρ is the total density, so that

$$\frac{dn_j}{dt}=\sum_{\alpha,\beta}K_{\alpha\beta}n_\alpha n_\beta+\sum_{\alpha,\beta,\gamma}K_{\alpha\beta\gamma}n_\alpha n_\beta n_\gamma-3n_j(8\pi G\rho/3)^{\frac{1}{2}}. \quad (56)$$

Consequently, we can write for neutrons the following rate equation:

$$dn_n/dt=An_pn_\nu-A'n_nn_{e^+}+Bn_pn_{e^-}-B'n_nn_\nu+Cn_pn_{e^-}n_\nu-C'n_n-3n_n(8\pi G\rho/3)^{\frac{1}{2}}. \quad (57)$$

Equation (57) can be rewritten using some of Eqs. (46)–(53) as

$$dn_n/dt=m_e^5c^{10}a_0[n_pK_p-n_nK_n]-3n_n(8\pi G\rho/3)^{\frac{1}{2}}, \quad (58)$$

where

$$K_p=I_A+I_B+I_C, \quad (58a)$$

and

$$K_n=I_{A'}+I_{B'}+I_{C'}. \quad (58b)$$

The limits of integration in the six integrals involved in K_p and K_n make it possible to combine certain pairs, with the result that

$$K_p=\int_1^\infty \frac{\epsilon(\epsilon^2-1)^{\frac{1}{2}}}{1+\exp[\epsilon x]}\left\{\frac{(\epsilon+q)^2\exp[\epsilon x]}{1+\exp[(\epsilon+q)x_\nu]}+\frac{(\epsilon-q)^2\exp[(\epsilon-q)x_\nu]}{1+\exp[(\epsilon-q)x_\nu]}\right\}d\epsilon, \quad (59)$$

and

$$K_n=\int_1^\infty \frac{\epsilon(\epsilon^2-1)^{\frac{1}{2}}}{1+\exp[\epsilon x]}\left\{\frac{(\epsilon+q)^2\exp[(\epsilon+q)x_\nu]}{1+\exp[(\epsilon+q)x_\nu]}+\frac{(\epsilon-q)^2\exp[\epsilon x]}{1+\exp[(\epsilon-q)x_\nu]}\right\}d\epsilon. \quad (60)$$

The equation describing the time rate of change of proton concentration can be written in a manner analogous to Eq. (58) as follows:

$$dn_p/dt = m_e^5c^{10}a_0[n_nK_n - n_pK_p] - 3n_p(8\pi G\rho/3)^{\frac{1}{2}}. \quad (61)$$

As shall be seen below, Eqs. (58) and (61) can be combined to give a single equation for the proton-neutron ratio, with the effect of the universal expansion not appearing explicitly.

The rate coefficients K_p and K_n have been evaluated numerically using Eqs. (59) and (60) for the range of values of x of interest. The values of x_ν corresponding to x have been taken from Table II, with[11]

$$q = 1 + (m_n - m_p)/m_e = 2.53.$$

It can be shown from Eqs. (59) and (60) that for small x, i.e., $x < 1$ where $x_\nu \to x$,

$$\lim_{x_\nu \to x} K_p = e^{-qx}K_n. \quad (62a)$$

Furthermore, for large values of x, one finds that

$$\lim_{x\to\infty} K_p = 0, \quad (62b)$$

and

$$\lim_{x\to\infty} K_n = \int_1^\infty \epsilon(\epsilon - q)^2(\epsilon^2 - 1)^{\frac{1}{2}}d\epsilon = 1.6318. \quad (62c)$$

The limit approached by K_n in Eq. (62c) for $x\to\infty$ is just the term $C'/(m_e^5c^{10}a_0)$ where $C'n_n = dn_n/dt$ describes free neutron decay and $C'(=\lambda)$ is the neutron-decay constant. One can, therefore, select the Fermi coupling constant g, which is the only undetermined constant in a_0 [see Eq. (44a)], so that the value of C' is the observed neutron decay constant.[12] The neutron half-life measured recently by Robson[13] is $\tau_{\frac{1}{2}} = 12.8 \pm 2.5$ minutes. The values of $m_e^5c^{10}a_0$ and of g corresponding to the measured half-life limits are given in Table III.

In Table I are given values of the dimensionless quantities K_n and K_p, evaluated numerically from

TABLE III. Values of Fermi constant for various neutron half-lives.

Neutron half-life (min)	10.3	12.8	15.3
$m_e^5c^{10}a_0$(sec^{-1})	4.627×10^{-4}	5.531×10^{-4}	6.876×10^{-4}
g(erg cm^3)	1.01×10^{-49}	1.11×10^{-49}	1.23×10^{-49}

[11] For $(m_n - m_p)c^2$ we have used 0.782 Mev, as given by D. M. Van Patter, Massachusetts Institute of Technology, Technical Report No. 57, January 1952 (unpublished), while m_ec^2 has been taken as 0.5110 Mev, as given by J. W. M. DuMond and E. R. Cohen, Phys. Rev. **82**, 555 (1951).

[12] Although the numerical constants in a_0 which depend on spin, etc., have been carefully evaluated in building up the rate coefficients, it may be noted that the equality $C'n_n/(m_e^5c^{10}a_0) = 1.6318$, with $C' = \lambda$ known from experiment, automatically yields a value for a_0, so that g, spin factors, etc., need not be separately specified.

[13] See reference 4. Recently L. M. Langer and R. J. D. Moffat, Phys. Rev. **88**, 689 (1952), obtained the value $\tau_{\frac{1}{2}} = 10.4 \pm 0.6$ min indirectly from studying tritium decay. This value and Robson's value agree within the probable errors.

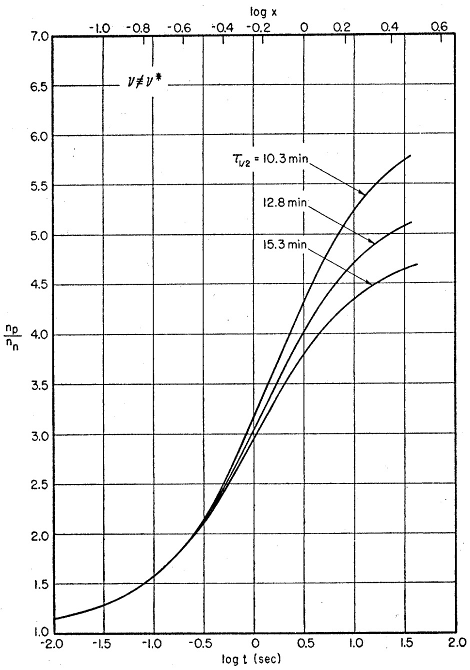

FIG. 2. The proton-neutron concentration ratio *versus* time and temperature ($x = m_ec^2/kT$) in the case of the Dirac neutrino (distinguishable neutrino and antineutrino) for the Robson neutron half-life value of 12.8 min, plus-and-minus the probable error.

Eqs. (59) and (60) in the range required for the present calculation. It may be noted that for x slightly greater than 3, K_n is close to the free decay value of 1.63 while K_p is negligibly small. Also for $x < 1$ the relationship $K_p \cong K_n \exp(-qx)$ holds quite closely.

Equations (58) and (61) describing neutron and proton concentrations can be combined by defining

$$\phi(t) = n_n/(n_n + n_p). \quad (63)$$

Taking the time derivative of ϕ and employing Eqs. (58) and (61) as required, one can write

$$d\phi/dt = m_e^5c^{10}a_0[K_p(1-\phi) - K_n\phi], \quad (64)$$

where a_0 is given by Eq. (44a). The actual integrations were done with $\ln x$ as independent variable where one writes

$$\frac{d\phi}{d\ln x} = \frac{d\phi}{dt}\frac{d\ln l}{d\ln x}\frac{dt}{d\ln l}. \quad (64a)$$

The quantity $d\ln l/d\ln x$ is given by Eq. (30) and calculated values are given in Table I, while values of $d\ln l/dt$, determined from Eq. (23), are also given in Table I.

Equation (64) has been integrated numerically for the six cases of interest, *viz.*, for $\nu \equiv \nu^*$ and $\nu \not\equiv \nu^*$ taking three values of the neutron half-life, namely, the mean value and the mean value plus-and-minus the probable error, as given by Robson.[4] The integration procedure was such as to give a final accuracy in ϕ of the order

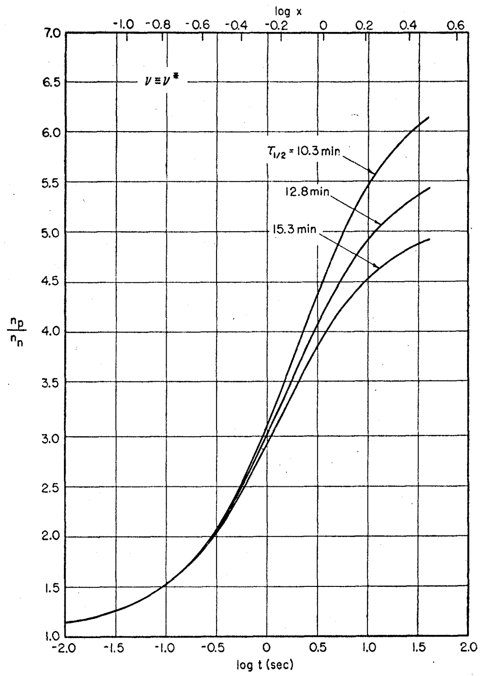

FIG. 3. The proton-neutron concentration ratio *versus* time and temperature ($x=m_ec^2/kT$) in the case of the Majorana neutrino (indistinguishable neutrino and antineutrino) for the Robson neutron half-life value of 12.8 min, plus-and-minus the probable error.

of the accuracy of the coefficients, i.e., $\sim$1 percent. The solutions we have obtained are presented in Figs. 2 and 3, where the proton-neutron ratio is plotted *versus* x and *versus* the time scale appropriate to the type of neutrinos involved. The integrations were carried from $x=0.1$ toward larger x, the initial value of $\phi=n_n/(n_n+n_p)$ being taken as the equilibrium value, i.e., $\sim[1+\exp(qx)]^{-1}$, since at $x=0.1$ the rate coefficients K_n and K_p are large and show negligible deviation from their respective equilibrium values. The integration interval was 0.1 in $\ln x$, with the first approximation at each step being the equilibrium value which Eq. (64) would predict at the given value of x. It may be noted in Figs. 2 and 3 that by $x=4$ further change in the proton-neutron ratio is almost entirely caused by free neutron decay.

A comparison of these results with those of Hayashi indicates that the major difference between our calculation and his may arise from the difference in neutron half-life used, *viz.*, Hayashi used 20.8 min, while the remainder of the differences, amounting to perhaps 20 percent in the proton-neutron ratio, arise from the use of relativistic quantum statistics, a more detailed cosmological model, and different temperatures for neutrinos and the rest of the medium in the present calculation. The proton-neutron ratio obtained by Hayashi by the time free neutron decay predominated was $\sim$4:1, whereas in the present calculation values from $\sim$4.5:1 to $\sim$6.0:1 are obtained depending on the half-life taken for the neutron and the type of neutrino considered.

It is interesting to note that if all the neutrons available at the start of element synthesis were used in making helium nuclei only, then the ratio of hydrogen to helium abundances corresponding to the range of proton-neutron ratios computed above would be from $\sim$7:1 to $\sim$10:1. Since some of the neutrons decay and some are involved in making the heavier elements, the above ratios would be minimum values of the initial universal H/He ratios. These values are consistent with the range of values obtained from astronomical data, *viz.*, $\sim$5:1 to $\sim$30:1 as found in planetary nebulae, stellar atmospheres, and theoretical stellar models.[14]

IV. DISCUSSION

In the preceding sections we have discussed quantitatively the physical conditions in the initial stages of the universal expansion. It now seems pertinent to mention some of the small physical effects whose influence on the present calculations has been neglected and also to comment on some of the limitations of the cosmological model when extrapolated to very early epochs.

The first question to be considered is whether or not the various processes, such as pair production, Compton and Coulomb scattering, etc., occur at sufficiently rapid rates to maintain equilibrium. A qualitative criterion as first described by Hayashi[3] is to compare the time required for the concentration of a constituent to change by about its own value with the time required for the universal temperature to change by about its own value. Characterizing these as relaxation times, τ, one finds from Eq. (6) and Table I that the relaxation time for temperature is given with sufficient accuracy for the present purpose by the following expression:

$$\tau_T\sim-\frac{dt}{d\ln T}=-\left(\frac{3c^2}{8\pi Ga_\gamma}\right)^{\frac{1}{2}}T^{-2}=2t. \tag{65}$$

To take a specific example one can calculate from the equilibrium concentration of electron pairs and their rate of production by photon-photon collisions the relaxation times, τ_{pair}, for pair production-annihilation. The result for $x\ll 1$ is

$$\tau_{\text{pair}}\sim-\frac{dt}{d\ln n_{\text{pair}}}\sim\frac{144x\hbar^3}{\pi^3m_ee^4}, \tag{66}$$

where e is the charge on the electron. Hence

$$\frac{\tau_{\text{pair}}}{\tau_T}\cong 10x^3\left(\frac{Gm^2}{\hbar c}\right)^{\frac{1}{2}}\left(\frac{e^2}{\hbar c}\right)^{-2}\sim 10^{-20}x^3\ll 1. \tag{67}$$

[14] A. Underhill, Symposium on the Abundance of Elements held at Yerkes Observatory, Williams Bay, Wisconsin, November 6–8, 1952 (unpublished), under the joint sponsorship of the National Science Foundation and the University of Chicago.

A similar result is obtained for other reactions not involving neutrinos, for which the change in the coupling coefficient $e^2/\hbar c$ does not greatly change the order of magnitude of the ratio. This is not the case for neutrino interactions. Hence all processes not involving neutrinos proceed at sufficiently rapid rates to maintain equilibrium.

The question of electron degeneracy is most easily examined by considering the requirement of electrical neutrality.[3] If one integrates Eq. (43) with a degeneracy parameter, ζ, included, then for high temperatures the electron or positron concentration can be written as

$$n_{e\mp}\sim(2/\pi^2)(kT/\hbar c)^3 e^{\pm\zeta}, \tag{68}$$

and

$$n_{e^-}+n_{e^+}\sim(4/\pi^2)(kT/\hbar c)^3\cosh\zeta. \tag{68a}$$

If the condition of electrical neutrality is imposed then $n_{e^-}-n_{e^+}=n_p$ and

$$\sinh\zeta=\frac{\pi^2 n_p x^3}{4(mc/\hbar)^3}\sim\frac{n_p}{n_{e^-}+n_{e^+}}. \tag{69}$$

As has been shown[1,2] the nucleon concentration during the early stages of the expanding universe is very small compared with the density of radiation $(1:10^6)$ and, therefore, also small compared with the electron-positron pair concentration. It follows then from Eq. (69) that the parameter ζ is very small and, therefore, the degeneracy of electrons or positrons properly has been neglected.

The charge on the electrons and positrons gives rise to a Coulomb interaction energy which contributes to the total energy of the medium. The reasonableness of neglecting this interaction energy can be seen from the following. The average distance between, say, electrons is found from Eq. (68), taking $\zeta\ll 1$ as

$$(1/n_{e^-})^{\frac{1}{3}}\sim\hbar/|\mathbf{p}|, \tag{70}$$

i.e., the de Broglie wavelength. It follows that the Coulomb energy, E_c, for two electrons is

$$E_c\sim-\frac{e^2}{(\hbar/|\mathbf{p}|)}=-\left(\frac{e^2}{\hbar c}\right)|\mathbf{p}|c\sim-\frac{1}{137}E_T, \tag{71}$$

where E_T is the mean thermal energy per electron. Because of this Coulomb interaction energy there will be a slight tendency for a given charge to have more nearest neighbors with charge of opposite sign. The fractional charge excess per nearest neighbor at the distance $\hbar/|\mathbf{p}|$ may be expected to be of the order $\exp[-E_c/kT]-1\sim 1/137$. Therefore, the contribution of Coulomb energy due to nearest neighbors to the total energy of the medium is $\sim E_c/137\cong E_T/(137)^2$ times the mean number of nearest neighbors. Assuming this number to be of the order of 10, the contribution of the Coulomb energy is $<10^{-3}E_T$, and can, therefore, be neglected.

The contribution of specifically nuclear forces is negligible because the nucleon density is very small compared with nuclear density. Furthermore, the energy evolution of nuclear reactions also can be neglected because it is itself small compared with the already small contribution of the low density of nucleons.

The foregoing small effects bear mainly on the cosmological model which has been discussed in Sec. II. There are also several questions of this kind which concern the calculation of the rates of the nuclear reactions in Eq. (37) which were determined in Sec. III. An examination of these reactions shows that of the six rates only $B'n_n n_\nu$ and $C'n_n$, Eqs. (48) and (53), involve two charged product particles. For these, one should more correctly include a factor in the reaction probability, w, to take into account the effect of Coulomb forces. In general this factor is given by[15]

$$2\pi\eta[1-\exp(-2\pi\eta)]^{-1}, \tag{72}$$

where

$$\eta=Ze^2E_e(\hbar c^2|\mathbf{p}_e|)^{-1},$$

Z is the nuclear charge, and E_e and $|\mathbf{p}_e|$ are electron energy and momentum, respectively. The effect of this correction on the two integrals in $C'n_n$ and $B'n_n n_\nu$ has been estimated and found to be less than one percent. Thus the effect of the Coulomb forces can be completely neglected in these cases.

As has been mentioned the matrix elements for the nuclear reactions stated in Eqs. (37) have been taken equal to unity for lack of a better estimate.[16] There seems to be little doubt that free neutron decay is a super-allowed transition since the decay rate is consistent with those of other light element β emitters. Furthermore, it would seem likely that the matrix elements for all the reactions considered here would remain about equal in the event that one included effects such as nucleon recoil.

It should also be pointed out that in calculating reaction rates we have considered that the nucleons, i.e., the heavy particles, are at rest. This approximation, which is customarily made, leads to a negligible error.

In addition to the above questions there are a number of more general points which may bear on the validity of the theory presented in this paper. One such question concerns the extrapolation of physical theories back to extremely high temperatures and densities. For example, some quantum field theories introduce a cutoff in, say, the electric field at the value it would have on the surface of the classical electron in order to avoid high-energy difficulties. This cutoff is introduced by appropriate modification of the field equations and, therefore, of the distribution of states in momentum space. When the mean electric field is equal to the

[15] See for example, G. Gamow and C. L. Critchfield, *Theory of Atomic Nucleus and Nuclear Energy Sources* (Clarendon Press, Oxford, 1949).

[16] E. Fermi, *Nuclear Physics* (University of Chicago Press, Chicago, 1950).

TABLE IV. Timetable of events in the early epochs of the expanding universe.

Temperature (Mev)	Remarks: Neutrino ≡ antineutrino	Remarks: Neutrino ≠ antineutrino
>100	Region of doubtful validity of the field equations where ρ_γ exceeds nuclear density.	
~100	Thermodynamic equilibrium prevails.	
	$\rho_\gamma \cong 1.2\times 10^{13}$ g/cm³ $\rho_\mu=(7/4)\rho_\gamma$, $\rho_\pi=(3/2)\rho_\gamma$ $\rho_\nu=(7/8)\rho_\gamma$, $\rho_e=(7/4)\rho_\gamma$ $t\cong 6.3\times 10^{-5}$ sec	Same as for $\nu\equiv\nu^*$ except $\rho_\nu=(7/4)\rho_\gamma$ $t\cong 5.9\times 10^{-5}$ sec
~100−~10	Mesons annihilate converting energy into photons, electrons, and neutrinos.	
~10	Neutrinos are freezing-in during this period.	
	$\rho_\gamma\cong 1.2\times 10^{9}$ g/cm³ $\rho_\mu\sim 10^{-6}\rho_\gamma$, $\rho_\pi\sim 10^{-6}\rho_\gamma$ $\rho_\nu=(7/8)\rho_\gamma$, $\rho_e=(7/4)\rho_\gamma$ $t\cong 8.7\times 10^{-3}$ sec	Same as for $\nu\equiv\nu^*$ except $\rho_\nu=(7/4)\rho_\gamma$ $t\cong 7.8\times 10^{-3}$ sec
~10−~2	Continued adiabatic expansion of universe with $T_\nu\cong T$ despite negligible interaction of neutrinos with medium.	
~2	Start of electron-positron annihilation.	
	$\rho_\gamma\cong 1.9\times 10^{6}$ g/cm³ $\rho_\mu=\rho_\pi\sim 0$ $\rho_\nu\cong(7/8)\rho_\gamma$, $\rho_e=(7/4)\rho_\gamma$ $t\cong 0.22$ sec	Same as $\nu\equiv\nu^*$ except $\rho_\nu\cong(7/4)\rho_\gamma$ $t\cong 0.20$ sec
~2−~0.05	Electron-positron annihilation, converting energy into photons. Neutrinos cool adiabatically relative to remaining particles, the latter maintaining thermodynamic equilibrium. [See Tables I and II for more details during this epoch.] The neutron-proton abundance ratio reaches the free decay value, 4.5:1−6.0:1, at $T\sim 0.2$ Mev. Nucleogenesis begins at $T\sim 0.2$ Mev.	
~0.05	Nucleogenesis is well under way.	
	$\rho_\gamma\cong 0.72$ g/cm³ $\rho_\nu\cong 0.24\rho_\gamma$, $\rho_e\sim 0$ $t\cong 600$ sec	$\rho_\gamma\cong 0.72$ g/cm³ $\rho_\nu\cong 0.47\rho_\gamma$, $\rho_e\sim 0$ $t\cong 550$ sec
~0.03	Nucleogenesis essentially complete except for charge adjustment by β decay.	
	$t\sim 30$ min	
~0.03 Mev −~1 kev	Thermonuclear reactions among some of the light elements, *viz.*, Li, Be, B, D with H, continue during this period.	
~0.015 ev	At $t\sim 10^8$ yr, $T\sim 170$°K and $\rho\sim 10^{-26}$ g/cm³, galaxies probably form.	

foregoing cutoff, one has

$$\sim[e/(e^2/m_ec^2)^2]^2\cong\rho_\gamma c^2,$$

which leads to a temperature of ~15 Mev. However, recent advances in quantum field theory obviate such a high-energy cutoff. In fact, if such a cutoff exists it is probably an order of magnitude higher. This is evidenced by the quantitative agreement between theory and the observed Lamb shift, for example. Cutoffs in momentum space must be larger than present day experimental energies, i.e., $kT>100$ Mev, since observed bremsstrahlung and pair production, etc., agree with theory quite well.

Another pertinent question is the possible contribution of equilibrium concentrations of "gravitational quanta" to the total density. Although at equilibrium the "graviton" density would be expected to be equal to the photon density, one must consider at what temperatures such equilibria can be maintained. We may apply Eq. (67) to the present situation and replace the coupling coefficient $(e^2/\hbar c)^2$ by the product of $(Gm^2/\hbar c)$ with an electronic or mesonic coupling coefficient whose value will be in the range $\sim 1-\sim 10^{-2}$. Then, since $(Gm^2/\hbar c)\cong 10^{-45}$ with $m=m_e$, one finds $\tau_{\mathrm{grav}}/\tau_T\sim 10^{22}$ at ~1 Mev. In order for $\tau_{\mathrm{grav}}/\tau_T\sim 1$, i.e., for the "gravitons" to maintain equilibrium, the temperature must be $\sim 10^4$ Mev.[17] It is difficult to see how the introduction of many-body processes would reduce this temperature drastically. One does not know how many different kinds of particles exist in the range $\sim 10^2-\sim 10^4$ Mev but on the basis of the presently known types of particles one can determine an upper limit to the graviton contribution. We can compute the ratio of graviton density to that of neutrinos down to the temperature at which neutrinos freeze-in, since beyond this temperature the ratio remains constant. From the analog to Eq. (35) one has, if $\mathfrak{F}_i$ represents degrees of freedom for each constituent present, i.e., $\mathfrak{F}_i\rightarrow\rho_i/\rho_\gamma$ as $T\rightarrow\infty$, the following relationship:

$$\frac{\rho_{\mathrm{grav}}}{\rho_\gamma}=\left(\frac{\sum_i \mathfrak{F}_i \text{ at } T_\nu'}{\sum_i \mathfrak{F}_i \text{ at } T_{\mathrm{grav}}'}\right)^{4/3}, \tag{73}$$

where T_ν' and T_{grav}' are the freeze-in temperatures of neutrinos and gravitons, respectively. From the presently known elementary particles which would exist at these temperatures one can estimate from Eq. (73) that $\rho_{\mathrm{grav}}/\rho_\gamma<0.1$ at T_ν'. During the subsequent expansion down to $T\sim 0.1$ Mev, $\rho_{\mathrm{grav}}/\rho_\gamma$ diminishes by a factor of ~4, just as ρ_ν/ρ_γ diminishes [see Table II]. At no time does the upper bound of the graviton contribution to the density exceed 2 or 3 percent, and the total contribution is probably much smaller.

Finally, it seems pertinent to comment on the question as to whether the density of nucleons relative to the density of radiation can be calculated at some very early time on the basis of theoretical considerations with complete symmetry between nucleons and antinucleons or whether it is a free initial condition. In particular can the nucleon density be the result of a statistical fluctuation in the competition between different processes of nucleon annihilation such as

$$p^+ + p^- \rightleftharpoons 2h\nu$$
$$\rightleftharpoons \pi^+ + \pi^-, \text{ etc.},$$

where p^+ and p^- are proton and antiproton, respec-

[17] It should be noted that in the coupling coefficient the quantity m must be taken to be the relativistic mass of the interacting particles, i.e., $(Gm^2/\hbar c)=(Gm_e^2/\hbar c)x^{-2}$. Also note that the numerical results given here for extreme physical conditions are at best rather crude approximations.

tively, and as yet unknown high-energy processes such as

$$p^+ \rightleftharpoons \text{mesons} + e^\pm, \text{ etc.?}$$

An examination of this question on statistical grounds yields a probable residual density of nucleons approximately equal to $\rho_\gamma/N^{\frac{1}{2}}$, where N is the number of nucleons initially present in any given finite volume under consideration in co-moving coordinates.[18] If we take an initial volume corresponding to the presently observable universe, the residual number of nucleons is found to be less than would be required to form the earth. It appears that the situation described above is untenable and that the initial nucleon concentration must be specified arbitrarily. This result is in agreement with present thinking in elementary particle physics which does not allow for single nucleon annihilation processes. In addition, it should be pointed out that no catalytic type of reaction (e.g., $2p^+ + p^- \rightarrow 2p^+ + \mu^-$) can vitiate the above statistical arguments because of the finite propagation velocity of disturbances noted in Sec. II.

V. CONCLUSION

The problem discussed in this paper has been concerned with the detailed nature of the general nonstatic homogeneous isotropic expanding cosmological model derived from general relativity as well as the elementary particle reactions which occur during early epochs. The study of the elementary particle reactions leads to a knowledge of the time dependence of the proton-neutron concentration ratio which is required in the problem of nucleogenesis. While the problem of element origin stimulated the present study, the results concerning the cosmological model are of interest in themselves. On the basis of the new physical conditions which have been discussed here, it would appear necessary to reexamine the specific reactions among the lighter nuclei, particularly as regards the missing species at $A=5$.

In order to summarize, we have presented the above calculations in abbreviated form as a timetable of events in the very early stages of the expanding universe, through the period of residual thermonuclear reactions[19] and galaxy formation.[20] In Table IV are given for various temperatures the corresponding epochs according to the expanding cosmological model involving the interconversion of matter and radiation, the densities of the various constituents according to the appropriate relativistic quantum statistics, as well as remarks concerning some of the principal physical phenomena that occur during these various early stages. This tabulation, it will be noted, covers both distinguishable and indistinguishable neutrinos.

Finally, we should like to point out that all of the results presented in this paper follow uniquely from general relativity, relativistic quantum statistics, and β-decay theory without the introduction of any free parameters, so long as the density of matter is very small compared with the density of radiation.

VI. ACKNOWLEDGMENTS

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[18] This can be seen from the following arguments. Let the numbers of protons, antiprotons, neutrons, and antineutrons in any finite co-moving volume V be equal and equal to N. Let α be the probability per particle for any of these particles to transmute to mesons at high temperature. We shall suppose that such transmutations occur first in the expansion, and that annihilation occurs later. This situation yields the largest residual density. Then on the average $4\alpha N$ particles transmute to mesons. The standard deviation σ in the number transmuting is then

$$\sigma = [4\alpha N(1-\alpha)]^{\frac{1}{2}},$$

which is a maximum of $N^{\frac{1}{2}}$ for $\alpha = \frac{1}{2}$. One may expect that in any volume V the excess of nucleons over antinucleons, or conversely, will be of the order of σ, i.e., of the order of $N^{\frac{1}{2}}$. The concentration of these residual nucleons at a later time when the initial volume V_0 has expanded to V_1 is given by $n_{\text{nuc}} = (N^{\frac{1}{2}}/V_0)(V_0/V_1)$. To a rough approximation the number of photons originally in V_0, a number approximately equal to N initially, has remained constant down to V_1, so that $V_0/V_1 \cong n\gamma_1/n\gamma_0$, where n_γ is photon concentration. Hence one can write

$$n_{\text{nuc}} N^{\frac{1}{2}} n\gamma_1/(V_0 n\gamma_0) \cong n\gamma_1/N^{\frac{1}{2}}, \quad \text{or} \quad \rho_{\text{nuc}} \sim \rho\gamma_1/N^{\frac{1}{2}}.$$

[19] Alpher, Herman, and Gamow, Phys. Rev. **74**, 1198 (1948).

[20] G. Gamow, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 10 (1953).